1. (15 points) If \( g = \gcd(a, b) \) then
   - show that \( g = ka + lb \), where \( k \) and \( l \) are integers.
   - List an algorithm which takes \( a \) and \( b \) as input and outputs \( g, k, l \) (such an algorithm goes by the name “extended euclidean” algorithm).
   - show that if \( g_1 = ua + vb, g_1 \neq 0 \), where \( u \) and \( v \) are integers, then \( |g_1| > |g| \).

2. (5 points) If \( a \equiv b \mod m \) and \( a \equiv b \mod n \) and \( \gcd(m, n) = 1 \), show that \( a \equiv b \mod mn \).

3. (5 points) Show that if \( \gcd(a, m) > 1 \), that \( a \) has no inverse mod \( m \).

4. (5 points) In the class we proved Fermats Little Theorem \( a^{p-1} \equiv 1 \mod p \). Give an alternate proof by induction on \( a \) (trivial for \( a = 1 \), prove that it is true for \( a + 1 \) if it is true for some \( a \)).